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An Exploration of Students' Conversions from a Symbolic to a Verbal Representation

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ABSTRACT This study was carried out with 48 pre-service mathematics teachers on a Real Analysis for Teachers course. The study was exploratory in nature with the purpose of understanding the challenges involved in changing a symbolic statement into a verbal written statement. The analysis of data generated from the students' written responses revealed that most students recognised and decoded the different parts of the symbolic statement correctly. However problems emerged at the syntax level, where many failed to arrange the words and phrases to create well-formed sentences. Although some students had developed familiarity with certain linguistic and symbolic resources, there were many students who could not distinguish between the universal and existential quantifiers. Instructors are advised to provide opportunities for students to interrogate the meaning and implications of statements provided in symbolic form before progressing to more complex proofs which utilise complex symbolisation

INTRODUCTION

Many researchers (Sfard 2008; Berger 2013) take the perspective that learning mathematics is a process of increasing participation in a mathematical community of practice. Sfard (2008: xvii) introduces the term 'commognition', which is a combination of communication and cognition and emphasises "that interpersonal communication and individual thinking are two facets of the same phenomenon". As newcomers communicate with more experienced participants, they begin to use the tools and resources of the domains more appropriately, and this enhances their participation as their practices become endorsed by the community. The role of language is central to communication during the learning of mathematics. Mathematics as a discourse entails different uses of language from informal verbal communicative resources to highly specialised symbolic forms. However, few studies (Dubinsky 1991; Duval 2006) have looked at how students are able to shift between different registers. This study is focused on this shift in registers by analysing students' written responses to tasks which required them to change symbolic statements to verbal statements. In addition the tasks required them to provide an illustrative example of the statement if it was true, and a counter example if it was not true. The examples chosen by the students are used to shed light on their own understanding of elements of the symbolic register as well as their conceptions of the real number system.

Literature Review

From Buchbinder and Zaslavsky (2009), we learn that generally mathematical statements can be classified into universal and existential, according to the quantifier that appears in the statement, which can sometimes be implicit. A universal statement asserts that a proposition is true for all values of the variable in the domain, while an existential statement asserts that there is an element in a domain for which the proposition holds true. Bardelle (2011) explored the difficulties related to negating a verbal statement with the universal quantifier with 202 undergraduate students. She found that lack of awareness of the functions of mathematical language negatively influences the use of logical operators and quantifiers. Bardelle (2011) noted that mathematical reasoning is affected by logical operations such as implication, negation, conjunctions as well as by the use of quantifiers. Dubinsky (1991) also identified the ability to cope with the quantifiers as a considerable barrier to understanding the usual formal definition of the limit. Bardelle (2011) found that the meanings assigned

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by students to negation of a sentence often do not correspond to mathematical language conventions but are affected by experiences from everyday life contexts. Colloquial common sense given to quantifiers and to linguistic expression with quantifiers formed a hindrance to students' conceptualisations.

In this study students were asked to determine whether two statements (one false, one true) were true or not. They were then asked to make up examples to illustrate or refute the given propositions. The word example is used in mathematics education in a variety of ways and offering an example describes a situation in which something specific is used to represent a general class (Bills et al. 2006). It is this linking role between the general and particular that makes an example an important resource in the teaching and learning of mathematics. The concept of a personal example space was formulated by Watson and Mason (2005) and describes a personal collection of accessible examples that is made available in certain situations and under certain conditions. Watson and Mason assert that there is little difference between an example and a counterexample; whether it is one or the other depends on where your attention is anchored. An example of a theorem could be a counterexample to an inappropriate version of the theorem. Buchbinder and Zaslavsky (2009: 226) use the term 'confirming example' to describe an example which satisfies a theorem or proposition. The authors express concern about the tendency of students to rely on specific examples as sufficient for determining that a general claim is true. These authors suggest that it is an incomplete understanding of the logical relations that lies at the heart of this tension.

Since the focus of this study is to explore the challenges of switching from a symbolic mathematical statement to its verbal equivalent, Duval's (2006) work on semiotic representations will provide some insight. Duval identifies the activity of conversion of representations from one semiotic representation system to another as one of the main sources of difficulties in learning mathematics. A conversion is a transformation of a representation which consists of changing a register without changing the objects being denoted (Duval 2006). One of the reasons put forward by Duval for the difficulty associated with conversions is that there are no rules or procedure that can be defined to encode or translate any conversion from one system to another, that would hold in various situations.. He used the example of encoding the verbal statement "the set of points whose ordinate is greater than their abscissa "into the symbolic representation "y >x".

The set of points whose ordinate is greater than their abscissa

$$y > x$$

When the verbal statement is changed slightly to "The set of points whose abscissa and ordinates have the same sign", then the appropriate symbolic statement is $x.y> 0$.

The procedure used to encode the first statement did not work for the second statement as a result of the slight change in the statement. A further transformation was made to the statement, before the encoding could be accomplished. This illustrates Duval's contention that there are no rules that can be used for conversions because slight changes to the object of the representation may require a different shift in the new representation.

Functional Linguistics and Mathematical Statements

In mathematics, patterned collective activity takes the form of mathematical discourse where a discourse is a special type of communication characterised by a range of permissible actions and reactions (Berger 2013: 1). Learning mathematics necessarily involves acquiring experience in participating in particular discourses. The process of participating in a mathematics discourse, involves creating narratives about objects in a new discourse. As we communicate with more experienced discussants, we begin to see how to use the elements of the new discourse appropriately, and we modify our use of these elements (Sfard 2008). Hence learning mathematics can be seen as a series of discursive shifts, which is enabled by corresponding shifts in the way the language is used. Hence the language adopted in communicating mathematics is a resource for the construction and negotiation of shared meanings that lead to mathematical knowledge (Planas and Civil 2013).

In order to examine the role of language use in deciphering mathematical statements, the researcher draws upon functional linguistics which is a perspective of language that focuses on the functions of text as opposed to form (Bardelle and Ferrari 2011). The system of language is 'instantiated' in the form of a text which has no meaning outside the system; a text in English for example has no semiotic standing other than by reference to the language system of English (Halliday 2004). In functional linguistics it is a register that forms the link between text and context. A register can be described as a functional variety of language related to use. It is "formed through the selection of resources available for a subject, related to his/her goals" (Bardelle and Ferrari 2011: 234). This study is focused on two registers set within the context of the real number system - the mathematical symbolic register and the verbal mathematical register. The study looks particularly at students' conversions of two statements from a symbolic representation to a verbal representation.

According to Halliday (2004), constituency in lexicogrammar means there is a scale of rank in the grammar in any language. In English it can be word, phrase, clause, sentence, where each consists of one or more ranks of the preceding one; for example, 'Come!' can be seen as a sentence with one clause consisting of one phrase with one word. Hence discursive ease in the particular register is demonstrated by the interpretation and production of texts using words that are combined to form phrases that can be combined to form clauses that are combined to form sentences. In the symbolic mathematics register, the unit is a symbol, which could signify operators, objects, processes, conjunctions etc. Symbols can represent words or phrases and these can be combined to form clauses which are combined to form propositions or compound statements. In a similar manner that we have words/phrases which may function as verbs, conjunctions, pronouns, adverbs etc., in the symbolic mathematics registers certain symbols or symbol phrases perform different functions such as connectives (conjunctions, disjunctions, conditional), objects (variables or expressions), operators (implies that, is equivalent to), quantifiers (existential, universal), etc. Discursive fluency in the register is demonstrated by the interpretation and production of compound statements or mathematical propositions made up by combining these different symbols and symbol phrases using the appropriate syntax (the arrangement of words and phrases to create well- formed sentences).

Communication in mathematics involves the adoption of several registers depending on whether the task requires informal explanations or discussions, a justification or proof, or a solution to a mathematical problem. In fact Duval (2006:3) maintains that the characteristic feature of mathematical activity is the simultaneous mobilisation of at least two registers of representation, or the possibility of changing at any moment from one register to another.

Hence mathematical communication may draw upon different linguistic resources and sometimes may require a switch between registers. In mathematics there are informal verbal registers used during communication by students and teachers to express ideas or engage in discussion about concepts; a formal mathematical verbal register used to communicate mathematical ideas in assessment settings and a more formal mathematical symbolic register made up of a combination of words and symbols as described in the preceding paragraphs. Here verbal may mean both written and spoken activities. The verbal mathematical register is also a specialised one comprising mathematical linguistic resources. Some of these terms and phrases differ from their everyday meaning; for example "irrational" describes a number that is not rational whereas in everyday use it could describe a decision that does not make sense.

METHODOLOGY

This qualitative study was exploratory in nature with the purpose of understanding the challenges involved in moving from a formal symbolic mathematics register to a verbal mathematics register. The participants were 48 preservice mathematics teachers enrolled in a Real Analysis course for teachers where students study topics in set theory, topology of the real line, number theory, countability, proof, and sequences and series. Many of these topics require immersion into the formal symbolic mathematics register, and students who do not develop the necessary fluency in the register struggle to understand the concepts whose descriptions can only be accessed by elements in the formal register. Hence the tasks in this study were designed to uncover barriers that students may experience when attempting to move between representations in the different registers, with the pedagogic aim of identifying possible interventions that could ease the process.

The questionnaire consisting of two tasks was administered to the participants. The analysis of the written responses can be viewed as content analysis which is to throw additional light on the source of communication and its producer (Cohen et al. 2000). Neuman (2011: 323) states that "the process of placing words, messages, or symbols in a text to communicate to a reader or receiver occurs without influence from the researcher who analyses its content." Dey (1993: 30) describes data analysis as "a process of resolving data in to its constituent components to reveal its characteristic elements and structure." In a similar manner the students' responses were broken down into constituent parts reflecting their engagement with the symbols and phrases used in the tasks. This was done in order to classify responses according to the interpretations or translations of the statements and to make connections across the data elements (Henning 2004). The responses were coded which means representing "the operations by which data are broken down, conceptualized, and put together in new ways" (Strauss and Corbin 1998: 120) in order to analyse their adoption of the symbolic register. Such an analysis took place on two levels — the actual words used by the respondents and the conceptualisation of these words by myself, the researcher.

Details of Tasks

The conversion in the two posed tasks (see Table 1) involves decoding from the symbolic register to a verbal register. Four demands related to the conversion can be distinguished. Firstly, the conversion involves the decoding of single symbols or symbol phrases into verbal phrases or clauses. The symbols and symbol phrases in Task 1 are $x \in \mathbb{R}$; $x \in \mathbb{Q}$ while those of Task 2 are $m \in \mathbb{N}$; $t \in$; $m<t<m+1$. R,ℕ and Q represent the set of real numbers, natural numbers and rational numbers respectively (Table 1).

Secondly it involves the decoding of quantifiers and connectives into verbal phrases. The connective in Task 1 is "$\Rightarrow$" (implies that) while that of Task 2 is such that. There are no quantifiers in Task 1 while those in Task 2 are the universal quantifier $\forall$ (for all) and the existential quantifier $\exists$ (there exists).

The third demand involves combining the decoded verbal phrases using the syntax rules to form a coherent mathematical proposition or statement which can be seen as a whole. A fourth demand is the interpretation of the proposition in order to make a judgement or evaluation about its applicability. In order to identify the extent to which the fourth demand was met, two further questions were added to each task. Students were asked to evaluate whether the statement was true or not, and to provide an illustrative example of the statement if it was true, or a counterexample if the proposition was not true. These demands are illustrated in Table 2 using Task 1.

RESULTS

The results for Task 1 are presented followed by those for Task 2. The presentation of the results for each task are organised in terms of the challenges encountered by the students in moving from the symbolic to the verbal register

Table 1: Tasks in study

Tasks	There are two statements given below a. Express the statements in 'English' sentences. b. Then say whether it is true or not true. c. If true, give an example that illustrates the statement. If it is not true, give a counterexample showing that it is not true.	
	1. $x \in \mathbb{R} \Rightarrow x \in \mathbb{Q}$	2. $\forall\, m \in \mathbb{N}\ \exists\, t \in \mathbb{Q} \ni m<t<m+1$
Translation	Every real number is rational	For every natural number n, there exists a rational number t, such that t lies between m and m+1
Comment	Note that this statement is not true, because there are real numbers which are irrational.	This statement is based on the density theorem which states that between any two real numbers it is always possible to find a rational, hence showing that the set of rational numbers is "dense" in the set of real numbers. The statement here is that the rationals are "dense" in the set of natural numbers and is true.

Table 2: Demands embedded in the conversion of the statement in Task 1

	$x \in \mathbb{R}$	$\Rightarrow$	$\chi\ \varepsilon\ \mathbb{Q}$
Decoding of symbols	x is an element of the set of real numbers		x is an element of the set of rational numbers
Decoding of connectives or quantifiers		Implies that	
Rephrasing the translated phrases and forming a coherent whole, using rules of syntax and grammar	If x is a real number	then it implies that	x is a rational number
Evaluation	Recognising that statement is false and providing a counterexample		

and in terms of the evaluation of the statement. The students are referred to as Student 1 (S1) up to Student 48 (S48), where the numbers reflect the order in which the responses were collected from the class.

Results for Task 1

Shifting From the Formal Symbolic Register to the Formal Verbal Register

The conversion process can be seen as a decoding of the symbolic statement $x \in \mathbb{R} \Rightarrow x \in \mathbb{Q}$ into a coherent verbal statement. There are two symbol phrases ($x \in \mathbb{R}$; $x \in \mathbb{Q}$), no quantifiers and a connective symbol ($\Rightarrow$)also referred to as the conjunction. The corresponding decoded verbal clauses would be *x is an element of the set of real numbers* and *x is an element of the set of rational numbers* and the conjunction phrase is "*implies that*". However the construction of a coherent verbal statement requires a reorganisation of the combination of the decoded verbal clauses.

It was found that all except one in the group of 48 were able to express the statement more or less correctly using the verbal representation.

S4 was the one who provided an incorrect conversion: *x is an element of real numbers if x is an element of rational numbers*. This student's error was an incorrect translation of the $\Rightarrow$ symbol to "if" instead of "implies that". This resulted in his statement being actually the converse of the symbolic statement. Perhaps the student responded in this manner because the symbolic statement is not true and in his translation he tried to replace the invalid statement by a valid one.

In deriving the verbal representation, some responses indicated problems with providing the exact translation of a single symbol or a symbol phrase, as in the case of S4 who incorrectly translated the symbol. Another example is that of S2 who provided an incorrect translation of x ε R as *for every real number of x*.

In certain cases, students correctly translated the symbols and/or symbol phrases into correct verbal clauses but were unable to identify appropriate conjunctions to link the clauses. In other cases they failed to rephrase the translated verbal clauses to make the 'whole' more coherent. Some examples of this are:

S1: *For all real values of x which also implies that x is also a rational number.*

S23: *If x is an element of real numbers that is implies that x is an element of rational number*.

S45: *If a real number x, it implies that it's a rational number*.

S1 correctly translated the symbol phrases into corresponding verbal clauses. However she failed to link the two clauses with an appropriate conjunction phrase and she also stuck rigidly to the exact connective phrase thereby disrupting the flow of the verbal statement. A clearer conversion statement is "For all real values of x, it is true that x is also a rational number." S23 started the first clause with "If", but did not insert a corresponding "then" before the second clause which resulted in the statement sounding disconnected. The response of S45 shows that the symbol phrases were correctly translated. However the conversion did not take into account syntax considerations. The conjunctions were unsuitable and the clauses needed to be rephrased to promote coherence across the clauses.

There were also some grammatical errors such as mixing up the plural and singular. For example:

S7: *Every real numbers then is rational numbers*

However this student displayed some discursive fluency in the verbal mathematical register because he expressed the statement differently using an equivalent statement. There were 27 students who started their description with the words "every real number" as opposed to the 19 who stuck to the more literal translation, "x is an element of the real numbers."

Evaluation of Statement

There were 14 students who correctly judged that the statement was not true, while 34 students asserted that the statement was true. The 34 students who produced the correct literal translation, but asserted that the statement was true, gave various reasons why they thought so. There were 25 students who chose a real number which was also rational as an example to illustrate the statement. For example:

S1: *Take a real number of x= 7.5, this is also rational.*

S6: *Because every real number can be written as a fraction for example, 5= 5/1.*

S1 was one of many (25) who chose a real number which was also rational as an illustrative example. Student S6 prefaced his example with an even stronger claim that every real number can be written as a rational number. His reasoning seems to be based on a definition of a rational number as a quotient of two real numbers, instead of the two integers.

There were three students who gave an example illustrating the converse of their statement, that is, that every rational number is real. For example:

S19: *Since all rational numbers fall under real numbers.*

S14 drew a Venn diagram illustrating that the set of rational numbers were a subset of real numbers, as illustrated in Figure 1.

It is clear from their responses that these students could not distinguish between the statement that they had correctly translated and its corresponding converse. They wrote a statement but provided explanations of the converse. A misconception related to notation was revealed by S4 who wrote

S4: $\mathbb{R}$ *= 100 and* $\mathbb{Q}$ *= 99.*

This response reveals a misunderstanding of mathematical convention of using symbols $\mathbb{R}$ and Q to represent the set of real numbers and the set of rational numbers respectively, while lower case letters *r* and *q* could be used to represent elements of these sets respectively.

On the other hand, there were 14students who correctly stated that the statement was false with five providing an appropriate counterexample, all of them using π as an example of a real number that is not rational.

Some students provided inappropriate counter examples such as

S42: *If x= 5.69243, then it is a real number however it is irrational rather than rational.*

The student cited 5.69245 as a real number which is irrational. There were similar responses by other students using the numbers 0.3; 5 and $\sqrt{-4}$ which they claimed were real and not rational numbers. Other students produced examples of real numbers which are rational such as

S16: *for example, if x=3, then x can be written as 3/1.*

Some responses revealed a misconception that the set of real numbers was separate from the set of rational numbers. For example:

S40: *Take three to be your real number, then it cannot be a rational number.*

S36: *x=2 is a real, not rational number.*

Both responses above indicate that if a number is real, then it cannot be rational, revealing

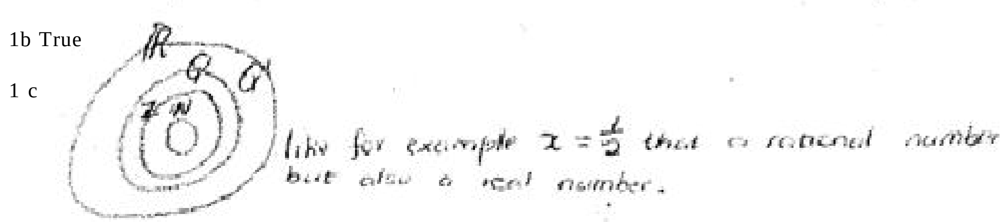

Fig. 1. Response by S14 using a Venn diagram

an underlying reasoning that the two sets were understood as separate.

Results for Task 2

Shifting from the Formal Symbolic Register to the Formal Verbal Register

The conversion process for Task 2 can be seen as a decoding of the symbolic statement $\mathbb{N}\exists t\in\mathbb{Q}\ni m<t<m+1$ into a coherent verbal statement. In Task 2 there are three symbol phrases: $m\in\mathbb{N}$; $t\in\mathbb{Q}$; $m<t<m+1$, which are decoded into the verbal clauses: *m is an element of the set of rational numbers; t is an element of the set of rational numbers* and *t is less than m+1 and greater than m* respectively. There are two quantifiers ($\exists$; $\forall$) which translate into the verbal clauses *there exists* and *for all* respectively. There is one connective symbol ($\ni$)which translates into the verbal conjunction *such that.* However the construction of a coherent verbal statement requires a reorganisation of the verbal clauses and conjunction phrases.

All 48 students were able to decode the symbolic statement more or less correctly into the verbal representation. That is all the responses conveyed the essence of symbolic statement, although there were some conversions that contained some errors. In certain cases, students translated symbol phrases incorrectly. In other instances, students correctly translated the symbol phrases, quantifiers and connective symbols but were unable to identify appropriate conjunctions to link the clauses. In some cases they failed to make the necessary rephrasing of the translated verbal clauses that would have resulted in a well formed sentence.

Some problems relate to the actual translations. The response of S9 below shows that the symbol phrase $m\in\mathbb{N}$ was not translated correctly:

S9: *For every natural numberin the element M, there exists a rational number in the element ksuch that t is less than m+1 and greater than m.*

In other cases students presented the correct translations of the symbol phrases but did not reorganise the verbal clauses to ensure a coherent verbal statement For example:

S4: *For all m is an element of natural numbers, there exists t is an element of rational numbers such that …..*

S2: *For all natural number m, there exists if t is an element of rational numbers such that m is less that t also t is less than m+1.*

S4 and S2 provided correct translations of the symbol phrases and quantifiers, but were unable to combine the clauses seamlessly to make a coherent statement. These students have struggled to string them together, although some like S4, may have even used correct connectives and conjunctions. Coherence requires the combination and even reorganisation of the translated verbal clauses to form a meaningful whole. For example S2 translated the symbol phrase $t\in\mathbb{Q}$ into *t is an element of rational numbers*, and translated the quantifierinto *there exists*, but failed at the syntax level which required a reorganisation within the verbal clauses. This re-organisation is a discursive skill that develops as people communicate by using the linguistic resources. As they develop this fluency they begin to interpret the phrases and the relationships connoted by the statement.

What also emerged was that certain students were comfortable with using equivalent forms of the symbolic clauses. A student who is able to present different verbal clauses to represent a symbol phrase displays more ease with the symbol phrase because h/she can see the equivalence between the exact translation and an equivalent one. An example of this is using the verbal clause, *Given any natural number m* as compared to the textbook translation, *For very m, which is an element of* $\mathbb{N}$*.*There were 27 students who used the first phrase instead of the textbook translation. Another indicator of discursive competence could be the translation of *m<t<m+1*into *t lies between m and m+1*, instead of the more literal translation *t is greater than m but is less than m+1.* There were 23 such students who provided the first translation, indicating that they were comfortable with the symbolic phrase.

Evaluation of Statement

All but one student correctly noted that the statement was true. S28, who stated that the statement was not true, wrote: *False, 25<126/5<26*. It seems as if he was trying to find a rational number between 25 and 26.

From those who agreed that the statement was true, there were three who did not give an illustrative example. For example S6 wrote:

S6: *Any number that is on the right hand side of the number line will be always greater.*

A total of eleven students provided an incomplete illustration by giving values of *m* but not a suitable value of *t* that would make the statement true. One such student was S4:

S4: *Take N to be 1000.*

The response of S4 reveals a misconception that $\mathbb{N}$ represented a number and not the set of natural numbers.

Other students' illustrations drew upon inappropriate values of *t*. For example:

S2: *Natural numbers = 1 and 2*
Rational number = $\frac{1}{2}$*/ 0.5*

There were 30 students who provided appropriate illustrations of the statement by drawing upon suitable values of *m* and *t*. For example:

S9: *Take m = 3 and t= 3,5. Then 3 < 3.5 < 4*

One of these students (S10) however, revealed a misconception of the number system when he wrote *t is a non-recurring decimal, therefore it is rational*. This student's response suggests that a recurring decimal is not rational.

DISCUSSION

The results show that there were only a few isolated cases of incorrect decoding of symbol phrases, connectives and quantifiers. Most problems emerged at the syntax level, where students generally struggled with linking the clauses together to form coherent well-formed statements. Duval (2006) alluded to the problem when he used examples of encoding two similar statement concerning relationships between the ordinate and abscissa of ordered pairs. The first encoding seemed to follow a direct translation rule with the placements of the symbols being directed by the chronology of the verbal descriptions. However when the statement was changed slightly to reflect a different relationship, it was not possible to follow the same steps that worked in the first example. Duval's point is that there are no translation rules for the process of encoding which could work for both statements. The second encoding required a reorganisation of the symbols so that it conveyed the meaning. Although this study looked at decoding the symbolic into verbal, it was found that the students were able to identify meanings of symbol phrases in the same way the students in Duval's study could link *x* to 'abscissa', *y* to 'ordinate', and > to 'greater than', but when it required a re-organisation of the verbal phrases, problems emerged. This re-organisation is a discursive skill that develops as students communicate using these linguistic resources. Fluency in the register is developed by using these resources and in turn as learners use the linguistic resources their understanding of the underlying concepts and their relationships is deepened.

The results reveal that students have developed familiarity with certain linguistic resources. For example there were 27 students who were able to describe the symbol phrase $x \in \mathbb{R}$ as *every real number* in Task 1 and 27 in Task 2 who described $m \in \mathbb{N}$ as *a natural number m* instead of the textbook translation, *m is an element of the set of natural numbers*, and the symbol phrase *m<t<m+1* was decoded by many as *t lies between m and m+1.* It seems as if these students have become accustomed to certain symbol phrases. Albano and Ferrari (2011) in their study which focused on communication which involved the adoption of different registers found that as students' fluency in the use of the registers improved, the students' use of the language became less rigid. This is a similar phenomenon observed here, the phrases that are widely used have been appropriated by the students as resources that are available and so students are not as rigid in translating them as they may be for other symbolic statements.

The study also revealed that some students who did not have well developed schema about the structure of the real number system. In fact most students did not recognise the statement from Task 1 as being false and only five students were able to provide a relevant counter example. Some students displayed a misconception that the sets of real numbers and rational numbers are separate. There were also misconceptions about what rational and irrational numbers are. Some students could not distinguish between notation used to denote the sets of natural numbers ($\mathbb{N}$), real numbers ($\mathbb{R}$) and rational numbers ($\mathbb{Q}$) as opposed to the notation used to denote elements of these sets.

The recognition of the validity or invalidity of the statement depended on the students' understanding of the number system. Providing illustrations or counterexamples was also dependant on the connections and repertoire of examples they had at their disposal or what is termed as their personal example space (Watson and Mason 2005). Many students opted for the same example of π as an irrational which is real. Goldenberg and Mason (2008) allude to a similar situation of students automatically producing π or $\sqrt{2}$ when asked for an example of an irrational number. This suggests that their accessible example spaces are limited to these examples of irrational numbers probably because they have not encountered many other examples of irrational numbers. These authors assert that as students are exposed to other examples, their example spaces will be extended.

Another problem seemed to be identifying the direction of the deduction as shown by some students who thought that the first statement was true but provided confirming examples of the converse of the statement.

An issue that emerged is the students' struggles with producing a counterexample to demonstrate that the proposition is not true. Some students did not seem to understand what it meant to negate the statement, as was reported in Bardelle (2011). The statement was $x \in \mathbb{R} \Rightarrow x \in \mathbb{Q}$, which is that all real numbers are rational. The negation of this universal statement is the existential statement that there exists some real number which is not rational. Hence the counterexample should have been an irrational number which is real. However some students produced examples of numbers like 3 and 5 showing that they were confused about the role of the examples. Bardelle attributed the difficulties experienced by students in negating the statement as emanating from their experiences of negating statements in real life which is different from that in mathematics. However this finding did not emerge in this study as a reason for the difficulty.

As seen in the results there were 25 students who stated that the statement was true and provided confirming examples for their assertion by choosing real numbers which were also rational. Note that the universal form of the statement (all real numbers are rational) is not true, however the existential form, 'there exists some real numbers which are rational' is true. It is likely that these 25 students did not recognise that the statement was a universal one and instead interpreted it as if it was the existential version, which is true. Hence the existential version may have appealed to them, instead of the incorrect universal statement that was given and their examples were illustrative of the existential version. This problem is not unique to these students as other studies have confirmed similar results (Bardelle 2011; Buchbinder and Zaslavsky 2009; Dubinsky 1991). The results indicate that much work is needed when introducing existential and universal statements to ensure that students understand the differences between the two.

CONCLUSION

The purpose of the study was to explore students' attempts at carrying out a conversion from a symbolic mathematics register to a verbal mathematics register. A first step of the conversion requires knowledge of the direct translation of individual symbolic phrases learning, which did not pose any problems for most students in this sample. However achieving discursive fluency requires fluent use of these mathematical linguistic resources. It is clear that even though students may recognise and identify symbols and symbolic phrases, they may not necessarily have access to the underlying meaning of the statements. Students also need to conjoin and reorganise phrases so that the proposition is coherent to other participants in the discourse, and most students struggled at this step. Furthermore, being able to create coherent statement is also not sufficient; it is necessary that the meaning of these propositions is conveyed by a judicious selection from a personal repertoire of examples to illustrate the proposition or provide contradictions to refute the proposition. Finding counter example to refute the statement from the first task proved most difficult with only five students being able to do this. There were 18 students who could not draw upon a suitable illustrative example to show the meaning of the second statement. The results further indicated that some students could not distinguish between the universal and existential form of a statement. The study shows that discursive fluency requires more than just recognition of the symbolic phrases and statements.

RECOMMENDATIONS

It will be appropriate that students are given opportunities to interrogate the meaning and implications of statements provided in symbolic form. Teachers should also provide such opportunities so that the gaps may be identified and addressed before moving on to even more complex symbolic propositions and theorems.

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